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Domatic Number in Cartesian Graph

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Abstract

A domatic partition of a graph G×H = (V, E) is a partition of V into disjoint sets $V_1, V_2, \ldots V_k$ such that each V_j is a dominating set for G×H. The maximum number of dominating sets, which the vertex set of a Cartesian graph G×H can be partitioned is called the domatic number of a graph G×H. It is denoted by dom(G×H) or d(G×H). In this paper, we discuss the sharp bounds for dom(G×H) and all Cartesian graphs attaining these bounds are characterized. We also describe the Cartesian product on complete graph G and H of order m and n and derive some properties and bounds on it.

Keywords: Dominating set, Domatic set, Domatic number, Cartesian graph, R-graph.

I. Introduction

A graph in this paper shall mean a simple finite, connected and undirected graph without isolated vertices. For a graph G=(V,E), V denotes its vertex set while E denotes its edge set, unless otherwise stated, the graph G=(p,q) has p vertices and q edges. Degree of a vertex v is denoted by deg(v). The complete graph on m vertices is denoted by K_m. The complement $\overline{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are adjacent in G. If S is a subset of V, then $\langle S \rangle$ denotes the vertex induced subgraph of G induced by S. The cardinality of a set S denoted by |S|, is the number of elements that S possesses. For all terminologies and notations in Graph theory we follow [1] and in particular all terminologies regarding trees we follow [2]. The concept of domination was first introduced by Ore[7] and C.Berge[6].

Definition 1.1

Let G = (V, X) be a graph. A subset $S \subseteq V(G)$ is said to be a dominating set [5] [11], if every vertex in V \ S is adjacent to atleast one vertex in S. The minimum cardinality taken over all minimal dominating sets is called the domination number of G and is denoted by $\gamma(G)$.

Many authors have introduced different types of domination parameters by imposing conditions on the dominating set and on the domatic number [9], [10]

Definition 1.2

The graphs G & H are complete graphs with the order of m and n, size of e_G and e_H respectively. The Cartesian product G×H of graphs G and H is a graph such that

- The vertex set of G×H is the Cartesian product V(G)×V(H) and
- Any two Vertices (u,u_1) and (v,v_1) are adjacent in G×H if and only if either u = v and u_1 is adjacent with v_1 or $u_1 = v_1$ and u is adjacent with v.

Every Cartesian graph is connected. Many variants of the domination number has been studied.

Definition 1.3

A graph G is said to be R-graph if $V(G) = V(K_{mn})$ and the adjacency of vertices should satisfy the following conditions,

i) The vertices in the same group should not be adjacent to each other.

ii) The first vertex of one group will be adjacent to all the vertices except the first vertex of the other groups. And the second vertex of one group will be adjacent to all vertices except the second vertex of the other groups and so on.

The purpose of this paper is to introduce the concept of domatic number in Cartesian graph.

II. Domaticity in Cartesian Graph

Definition 2.1

A domatic partition of a graph G = (V, E) is a partition of V into disjoint sets $V_1, V_2, \ldots V_k$ such that each V_j is a dominating set for G. The maximum number of dominating sets which the vertex set of a graph G can be partitioned is called the domatic number of a graph G, and it is denoted by dom(G) or d(G).

Proposition 2.2

If G and H are complete graph with m_1 and m_2 vertices then dom (G×H) = dom ($\overline{G\times H}$) = m_1.

Proposition 2.3

For any Cartesian graph G×H (which is formed by the complete graph G and H)

dom (R- graph) = dom ($\overline{G\times H}$)

III. Characteristics of Domatic Number in Cartesian Graph

Proposition 3.1

For any Cartesian graph (which is formed by Cartesian product of complete graph)

i) deg (G×H) = deg G + deg H
ii) deg ($\overline{G\times H}$) = (n – 1) deg G.

Proposition 3.2

For any Cartesian graph G×H (which is formed by Cartesian product of the complete graph G and H)

i) deg (G×H) = $m_1 + m_2 - 2$
ii) deg (G×H) ≤ deg ($\overline{G\times H}$)

Proposition 3.3

In Cartesian graph G×H, if H is a path then dom G×H = deg ($\overline{G\times H}$) + 1.

Proposition 3.4

For any Cartesian graph (which is formed by the complete graph G and H)

i) $\overline{G\times H}$ ≅ R-graph
ii) E($\overline{G\times H}$) = E (K_{mn}) \ E(G×H)

Theorem 3.5

For any complete graph G and H of order m and n. Let G×H be a Cartesian graph of order mn = m×n. d(G×H) = m iff G×H = K_{mn} \ R- graph.

Proof:

Let G and H be a graph of order m and n. Sufficiency, if G×H ≅ K_{mn} \ R- graph then by the Proposition 3.4(i), $\overline{G\times H}$ ≅ R- graph. Hence the vertex sets, the edge set of $\overline{G\times H}$ and R- graph are equal. Further the adjacency character also holds by proposition (2.3).

dom (R- graph) = dom (spanning tree) = m

∴d($\overline{G\times H}$) = m

Since $\overline{G\times H}$ ≅ R- graph.

Hence every minimal dominating set of G×H must contain atmost n vertices of V(G×H) .

∴ d (G×H) = m

Necessary condition, if m = 1 and m ≥ n, G×H ≅ E(K_{mn}) \ 0 edges of the R-graph.

Therefore, G×H ≅ E (K_{mn}). Suppose that S is a dominating set of G×H with side n.

If n ≥ 2, d (G×H) = m then in every minimal dominating set must contain n number of vertices, therefore mn-d \ S_i number of vertices lies in V \ S_i , that is outside of S_i.

We may assume that $S_i = \{(u_1,v_j)\}_{j=1 \text{ to } n}$.

∴ (u_2,u_3) is one of the vertices lie outside of in any one of S_i. Since d is a domatic set of G×H, (u_2,v_3) must be joined to some vertex in any number of S_i but not all vertices in any one of S_i. Therefore there exists vertices $(u_1,v_j)_{j=1 \text{ to } n}$ where i = 1, 2, …..m vertices must be joined to some vertices in any one of S_i.

Let (u_1,v_1), $(u_1,v_2) \in S_i$ for all i = 1, 2, ……..m and (u_2,v_2) be a vertex outside of S_i .

Therefore (u_1,v_1), (u_2,v_2) ∈ E (G×H) but $(u_1,v_1),(u_2,v_2) \in$E(G×H). {by definition of edge set of Cartesian graph}

As S_i is a dominating set of G×H and d_i is a domatic set of G×H which contain all S_i. There exists a spanning tree T of the induced subgraph ⟨S_i⟩.

Let $(u_1,v_3) \in S_i \setminus \{(u_1,v_2)\}$ be a leaf of T.

Since, $S_i \setminus \{(u_1,v_2)\}$ is not a dominating set of a G×H. Further $(u_i,v2)$, i =1, 2,3,… m vertices do not join with any one vertices of S_i. Hence $S_i \setminus \{(u_1,v_2)\}$ cannot be a

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dominating set of G×H, which implies that (u_1,v_2) is not joined to any vertex of $S_i \setminus \{(u_1,v_2)\}$ in G×H. Since there is no minimal dominating set of S.

Hence d is domatic of G×H which contain all S_i of G×H.

Let T_1 be a R-spanning tree and T' be the sub graph induced by all edges incident with (u_1,v_2) in $\langle S_i \rangle$. It is clear that T' is a spanning tree of $\langle S_i \rangle$ with n-2 leaves. Take any leaf $(u_1,v_3) \neq (u_1,v_1)$ on T'. By similar argument to the above, we can derive that (u_1,v_3) is joined to all vertices of $S \setminus \{(u_1,v_3)\}$.

Thus $\langle S \rangle \cong K_n$ { Dejan Delic and Wang [10] } and also $T_1 \cong T'$.

To complete our proof, we only need to show that the $E(G\times H) \cong E(K_{mn}) \setminus$ (R-graph).

By the definition of Cartesian graph G×H contains only m × n number of vertices and ne_G+ne_H edges. From proposition (3.3),

we know that $E(G\times H) = E(K_{mn}) \setminus E(G\times H)$

$E(\text{R-graph}) = E(K_{mn}) \setminus E(G\times H)$

$E(G\times H) = E(K_{mn}) \setminus E(\text{R-graph})$ -------------- (1)

By the definition of R-graph and R-graph is derived only from complete graph, the vertex set of K_{mn} and R-graph are the same.

∴ From equation (1), the edge set of G×H must be equal to K_{mn} graph and R-graph.

It is clear that the R-graph and the complement of G×H are isomorphic. Hence they have the same vertex set.

From the definition of R- graph, $V(K_{mn}) = V(\text{R-graph})$.

Hence, $V(G\times H) = V(\overline{G\times H}) = V(\text{R-graph}) = V(K_{mn})$

∴ $V(G\times H) = V(\text{R-graph}) = V(K_{mn})$ ------------------ (2)

Further an equal number of vertices with a given degree.

From equation (1) and (2) $G\times H \cong K_{mn} \setminus$ R-graph.

IV Example

G

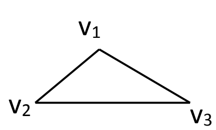

Figure 4.1 Complete graph K_3 = G

H

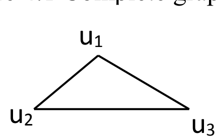

Figure 4.2 Complete graph K_3 = H

G×H

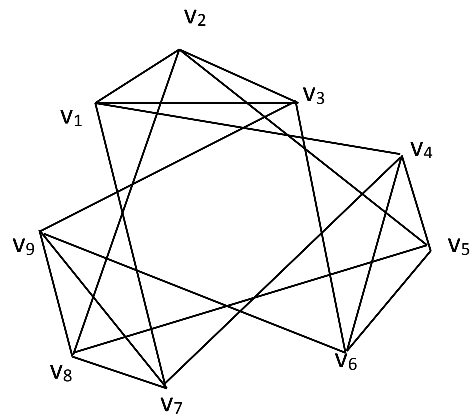

Figure 4.3 Cartesian product of G and H

$\overline{G\times H}$

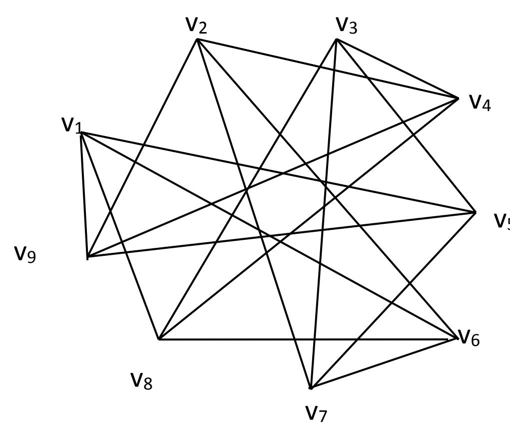

Figure 4.4 Complement of G×H

R-graph

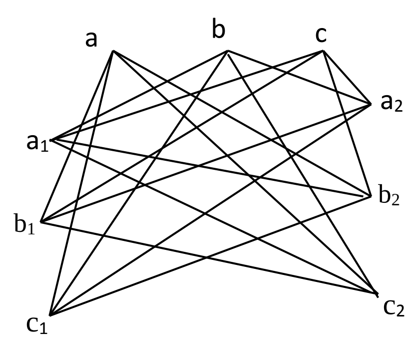

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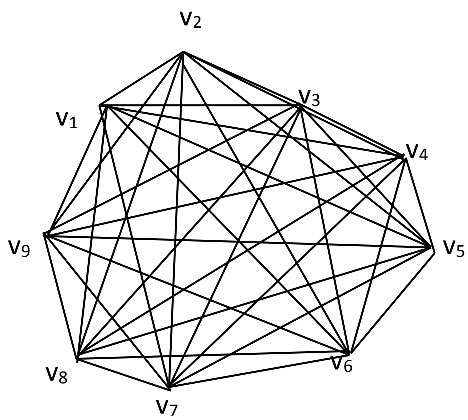
Figure 4.5 R-graph with 3 grouping vertex set

K_{mn}



deg(G)=2, **deg (H)=2**
deg($G\times H$) = 4, **deg($\overline{G\times H}$) = 4**
deg(R-graph) = 4, **deg(K_{mn}) = 8**
dom($G\times H$)=3, **dom(K_{mn}) = 8**
dom($\overline{G\times H}$) = 3.

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